Knowledge Representation and Reasoning, WS 17/18
Probeklausur

Surname, First Name: .................................................................

Matriculation number: ..............................................................

Signature: ..............................................................................

Organisational Hints:

1) At first, please fill out the form above.
   Check the sheets for completeness (Sheet 0, 1, ..., 13) and the print image for correctness.
   Have the student ID card and a photo ID card ready for inspection.

2) No tools except stationary material are allowed.
   Please turn mobile phones off.

3) No loose sheets. Keep this stack of sheets together.
   Only those solutions are evaluated, which are written on the sheets of this stack.
   There is enough space for all answers. If you need more space, you can use the back sides and the empty auxiliary sheets 12–13. In these cases, please refer in the question to the place where the answer can be found.
   If necessary, you can ask the supervisor for more blank auxiliary sheets.

4) Your answers may be in English or in German.
   The answers must convince the referee that you really understood the subject.

5) The text (... lines) in the questions gives the number of machine-written lines in the proposed solution. It helps you to estimate how detailed the answer is expected.


<table>
<thead>
<tr>
<th>question</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>∑</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximal points</td>
<td>18</td>
<td>10</td>
<td>16</td>
<td>10</td>
<td>7</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This examination paper is not to be evaluated.

Signature:

All questions are equally weighted. Please do not fill this part yourself ; - )

Mark:
Question 1  Canonical Model  

Consider the following facts:

- Farmer Krause owns at least 2 black horses.
- Farmer Krause owns at least 2 white horses.
- Farmer Krause owns at most 3 horses (with some colour)

1.a) Formulate these facts in $ALC$ with number restrictions. (3 lines)

1.b) Expand one open branch in the tableau calculus. (24 lines)

1.c) List the canonical model for this branch. (4 lines)

Solution:

1.a)

1  Krause : $\textit{atleast} \ 2 \ \textit{owns}$ (Horse $\sqcap \exists \textit{has-Colour} \ \textit{black}$)
2  Krause : $\textit{atleast} \ 2 \ \textit{owns}$ (Horse $\sqcap \exists \textit{has-Colour} \ \textit{white}$)
3  Krause : $\textit{atmost} \ 3 \ \textit{owns}$ Horse

1.b)

4  Krause : $\textit{owns} \ P_1$
5  $P_1 : \textit{Horse}$
6  $P_1 : \exists \textit{has-Colour} \ \textit{black}$
7  Krause : $\textit{owns} \ P_2$
8  $P_2 : \textit{Horse}$
9  $P_2 : \exists \textit{has-Colour} \ \textit{black}$
10  Krause : $\textit{owns} \ P_3$
11  $P_3 : \textit{Horse}$
12  $P_3 : \exists \textit{has-Colour} \ \textit{white}$
13  Krause : $\textit{owns} \ P_4$
14  $P_4 : \textit{Horse}$
15  $P_4 : \exists \textit{has-Colour} \ \textit{white}$
16  $P_1 = P_3$
17  $P_1 : \exists \textit{has-Colour} \ C_1$
18  $C_1 : \textit{black}$
19  $P_2 : \exists \textit{has-Colour} \ C_2$
20  $C_2 : \textit{black}$
21  $P_4 : \exists \textit{has-Colour} \ C_3$
22  $C_3 : \textit{white}$
23  $P_4 : \exists \textit{has-Colour} \ C_4$
24  $C_4 : \textit{white}$

17  $P_1 : \exists \textit{has-Colour} \ C_1$
18  $C_1 : \textit{black}$
19  $P_2 : \exists \textit{has-Colour} \ C_2$
20  $C_2 : \textit{black}$
21  $P_4 : \exists \textit{has-Colour} \ C_4$
22  $C_3 : \textit{white}$
23  $P_4 : \exists \textit{has-Colour} \ C_4$
24  $C_4 : \textit{white}$

1.c) Canonical Model:

Domain: \{Krause, $P_1, P_2, P_3, C_1, C_2, C_3, C_4$\}

Concepts: Horse: \{$P_1, P_2, P_4$\}, black: \{$C_1, C_2$\}, white: \{$C_3, C_4$\}

Relations: owns : \{(Krause, $P_1$), (Krause, $P_2$), (Krause, $P_3$)\}

has-Colour : \{(P_1, C_1), (P_1, C_3)(P_2, C_2)(P_4, C_4)\}
Question 2  Soundness Proof  

(2+8=10 Points)

2.a) Formulate the atleast-Rule for $\mathcal{ALC}$ with unqualified number restrictions.

2.b) List the part of the soundness proof for the tableaux calculus for $\mathcal{ALC}$ which deals with this rule.  

(4 lines)

Solution:

2.a) 

\[
\begin{align*}
\text{atleast } n &\quad r \\
x &\quad y_1 \\
\vdots &\quad \text{UNA}(y_1, \ldots, y_n) \\
x &\quad y_n 
\end{align*}
\]

2.b) Suppose an interpretation $\mathcal{I}$ satisfies $x : \text{atleast } n \ r$.

In this case there are different $v_1, \ldots, v_n$ in $\mathcal{I}_D$ with $x \mathcal{I} r v_i$ for $i = 1, \ldots, n$.

Define $y_i^\mathcal{I} = v_i$ for $i = 1, \ldots, n$. Then $\mathcal{I}$ satisfies $x : r y_i$ for $i = 1, \ldots, n$.

Since the $v_i$ are all different, the Unique Name Assumption holds.
Question 3  Concrete Domain  

(6+10=16 Points)

3.a) Model the following facts in \(ALC\) with Concrete Domain:

- A rich man is man who owns a house worth at least 1000000€ and a car with maximum speed at least 250 km/h.
- Karl is a man
- Karl owns VillaSunshine, which is worth 1500000€, and the car Herbie with maximum speed of 300 km/h.

3.b) Prove with tableaux calculus for \(ALC\) with Concrete Domain that Karl is a rich man.

Solution:

3.a)  
   a) \( \text{RichMan} = \text{Man} \sqcap \exists \text{owns} (\text{house} \sqcap \text{value} > 1000000) \sqcap \exists \text{owns} (\text{car} \sqcap \text{speed} > 250) \)
   b) Karl: \( \text{Man} \)
   c) Karl: \( \text{owns VillaSunshine} \)
   d) VillaSunshine: \( (\text{house} \sqcap \text{value} = 1500000) \)
   e) Karl: \( \text{owns Herbie} \)
   f) Herbie: \( (\text{car} \sqcap \text{speed} = 300) \)

3.b) Proof by Contradiction: (with some shortcuts)
   Karl: \( \neg \text{RichMan} \)
   Karl: \( \neg (\text{Man} \sqcap \exists \text{owns} (\text{house} \sqcap \text{value} > 1000000) \sqcap \exists \text{owns} (\text{car} \sqcap \text{speed} > 250)) \)
   Negation Normal Form:
   Karl: \( \neg \text{Man} \sqcup \forall \text{owns} (\neg \text{house} \sqcup \text{value} \leq 1000000) \sqcup \forall \text{owns} (\neg \text{car} \sqcup \text{speed} \leq 250) \)
   Case 1: Karl: \( \neg \text{Man}: \text{Contradiction with b).} \)
   Case 2: Karl: \( \forall \text{owns} (\neg \text{house} \sqcup \text{value} \leq 1000000) \)
   VillaSunshine: \( (\neg \text{house} \sqcup \text{value} \leq 1000000) \quad (\forall\text{-rule}) \)
   Case 2a: VillaSunshine: \( \neg \text{house}: \text{Contradiction with first part of d) \)
   Case 2b: VillaSunshine: \( \text{value} \leq 1000000 \)
   VillaSunshine: \( \text{value} = x \sqcap x \leq 1000000 \)
   VillaSunshine: \( \text{value} = y \sqcap y = 1500000 \) (second part of d)
   \( x = y \) (functionality of value)
   \( x \leq 1000000 \sqcap x = 1500000 \) is a contradiction.

   Case 3: Karl: \( \forall \text{owns} (\neg \text{car} \sqcup \text{speed} \leq 250) \)
   Herbie: \( (\neg \text{car} \sqcup \text{speed} \leq 250) \)
   Case 3a: Herbie: \( \neg \text{car} \)  (Contradiction with first part of f)
   Case 3b: Herbie: \( \text{speed} \leq 250 \)
   Herbie: \( \text{speed} = x \sqcap x \leq 250 \)
   Herbie: \( \text{speed} = y \sqcap y = 300 \) (second part of f)
   \( x = y \) (functionality of speed)
   \( x \leq 250 \sqcap x = 300 \) is a contradiction.
Question 4  Reflexivity  

Some binary relations are reflexive, for example `likes` (everybody likes himself). Reflexivity of a relation like `likes` can be easily incorporated into the Tableau Calculus for \textit{ALC} by adding in every branch for every constant symbol $x : \text{likes}(x, x)$. This adds a vast amount of irrelevant information, for example `likes(my\_computer, my\_computer)`.

\begin{enumerate}
\item[a)] List a small \textit{ALC}-example where reflexivity of a relation is needed to answer a query. \hspace{1cm} (2 lines)
\item[b)] Find a more efficient way of incorporating the reflexivity of a relation into the original Tableau Calculus for \textit{ALC}, by adding a single rule which adds only really relevant information. \hspace{1cm} (3 lines)
\item[c)] Prove the soundness of your new rule. \hspace{1cm} (2 lines)
\end{enumerate}

Solution:

\begin{enumerate}
\item[a)] Karl : $\forall$ \text{likes} clever\_Person  
Query: Karl : clever\_Person ?
\item[b)] The new rule is:

\[
\begin{array}{c}
\frac{x : \forall r \varphi}{x : \varphi}  \\
\text{if } r \text{ is reflexive}
\end{array}
\]

\item[c)] We must prove that if $x : \forall r \varphi$ is true in a model, then $x : \varphi$ is also true in the model. If $r$ is reflexive then $r(x, x)$ holds, and therefore $x$ is one of the $r$-successors of $x$, for which $\varphi$ is assumed to be true.
\end{enumerate}
Question 5  Open World Assumption  

5.a) What is the Open World Assumption?

5.b) In which of the following systems does the Open World Assumption hold? Answer with yes or no.

- ALC
- Datalog
- OPS5
- Prolog
- SWRL

Solution:

5.a) More facts may hold in the real world than in the computer model.

5.b)

<table>
<thead>
<tr>
<th>System</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALC</td>
<td>yes</td>
</tr>
<tr>
<td>Datalog</td>
<td>no</td>
</tr>
<tr>
<td>OPS5</td>
<td>not relevant</td>
</tr>
<tr>
<td>Prolog</td>
<td>no</td>
</tr>
<tr>
<td>SWRL</td>
<td>yes</td>
</tr>
</tbody>
</table>
Question 6  Datalog  

6.a) Explain the notion 'stratification' in Datalog. Where and why is it necessary?  (3 lines)

6.b) Explain the notion 'range restriction' in Datalog. Is it really necessary?  (3 lines)

Solution:

6.a) 'stratification':
   It is necessary if negation by-failure is to be used. In order to be able to evaluate a rule \textit{head} :- \textit{not p} without running into an endless loop, it must be possible to derive all consequences with \textit{p} without directly or indirectly using the same rule with the \textit{head}.

6.b) 'range restriction':
   It means that all variables in the head of a rule also occur in the body. If this is not the case, literals with variables could be derived. This is possible in principle, but requires a database which can handle facts with variables. It would require significant changes to current implementations.
Question 7  Fuzzy Controller

Consider a fuzzy controller for a fan control. The input data for the controller are the temperature (0 – 40°) and the humidity (0-100%). The output of the controller is the speed for the fan (0 - 1000 rpm).

The linguistic variables are low, weak, medium, high.

The Fuzzy values of Temperature are

```
<table>
<thead>
<tr>
<th>Temperature</th>
<th>Low</th>
<th>Weak</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0.85</td>
<td>0</td>
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<td>15</td>
<td>0</td>
<td>0</td>
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<td>0.85</td>
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<td>20</td>
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<td>0.85</td>
<td>0.85</td>
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<td>0.85</td>
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</tr>
<tr>
<td>30</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>35</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>0</td>
<td>0.85</td>
<td>0</td>
</tr>
</tbody>
</table>
```

The Fuzzy values of Humidity are

```
<table>
<thead>
<tr>
<th>Humidity (%)</th>
<th>Low</th>
<th>Weak</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.85</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0.85</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>0.85</td>
<td>0.85</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
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<td>40</td>
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<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>50</td>
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<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>60</td>
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<td>0.85</td>
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<td>70</td>
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<td>80</td>
<td>0</td>
<td>0.85</td>
<td>0.85</td>
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</tr>
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<td>100</td>
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<td>0.85</td>
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<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
</tbody>
</table>
```
The fuzzy values for Speed need only three linguistic variables.

![Fuzzy Value Graph]

The following rules are given (T: Temperature, H: Humidity, S: Speed)

<table>
<thead>
<tr>
<th>Rule</th>
<th>T</th>
<th>H</th>
<th>S</th>
<th>Value</th>
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<tr>
<td>1</td>
<td>low</td>
<td>low</td>
<td>low</td>
<td></td>
</tr>
<tr>
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<td>low</td>
<td>weak</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>low</td>
<td>medium</td>
<td>medium</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>low</td>
<td>high</td>
<td>medium</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>weak</td>
<td>low</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>weak</td>
<td>weak</td>
<td>medium</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>weak</td>
<td>medium</td>
<td>medium</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>weak</td>
<td>high</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>medium</td>
<td>low</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>medium</td>
<td>weak</td>
<td>medium</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>medium</td>
<td>medium</td>
<td>high</td>
<td></td>
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<td>high</td>
<td></td>
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<td>low</td>
<td>medium</td>
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<tr>
<td>16</td>
<td>high</td>
<td>high</td>
<td>high</td>
<td></td>
</tr>
</tbody>
</table>
Questions

4a) Describe two different strategies for combining the results of the rule applications. (1 lines)

4b) Describe two different strategies for computing the final speed-value (defuzzification). (2 lines)

4c) Choose a reasonable strategy for combining the results of the rule applications, and choose two different strategies to compute the final speed (RPM).

Compute the final speed (RPM) for the two input values for the controller:

Temperature: 17°
Humidity: 50%
You can use the above rule table for intermediate values during the computation. Use the Speed- figure above for estimating the finite value. Indicate how it is computed.

Solution:

4a) minimum, or average

4b) Strategies for computing the final Speed-value:
- mean of maximum (the middle of the largest plateau)
- center of gravity

4c) 1. Fuzzification:
   Temperature 17° low = 0, weak = 0.6, medium = 0.4, high = 0
   Humidity 50% low = 0, weak = 0.5, medium = 1, high = 0.

<table>
<thead>
<tr>
<th>Rule</th>
<th>T</th>
<th>H</th>
<th>S</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>low</td>
<td>low</td>
<td>low</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>low</td>
<td>weak</td>
<td>low</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>low</td>
<td>medium</td>
<td>medium</td>
<td>0</td>
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<tr>
<td>4</td>
<td>low</td>
<td>high</td>
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<td>0</td>
</tr>
<tr>
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<tr>
<td>16</td>
<td>high</td>
<td>high</td>
<td>high</td>
<td>0</td>
</tr>
</tbody>
</table>

Results of rule-applications with 'average'-strategy:

low  0
medium 1.4 / 7 ~ 0.2
high 0.4 / 5 ~ 0.1

Defuzzification
Results:
mean-of-maximum: 500 RpM
center-of-gravity: ~ 600 RpM
Question 8  Arc-Consistency  

8.a) Make the following constraint-network arc-consistent (the final result is sufficient)

8.b) List a concrete solution of this problem.

Solution:

8.a)

8.b) A:1, B:2, C:3, D:1, E:2,F:1,G:1,H:3