Exercise 5-1  Loopcheck

a) Give a precise definition of the $\rightarrow_{\exists}$-rule with loopcheck.

b) Check the consistency of $A$ given the axiom $A \sqsubseteq \exists r \top \sqcap \forall r (A \sqcap B)$

c) Check the consistency of $\text{Node} \sqcap \text{Edge}$ given the axioms

- $\text{Edge} \sqsubseteq \exists \text{ touches Node}$
- $\text{Node} \sqsubseteq \forall \text{ touches Edge}$

Exercise 5-2  Qualified Number Restrictions: Bon Appetit!

Consider the concepts $\text{Food}$, $\text{Spice}$ and the relation $\text{contains}$. In addition we have the following information.

- Mensa-Food is a Food not containing more than 2 Spices.
- Curry is a Mensa-Food with at least 5 Spices.

a) Formulate the above information as $\mathcal{ALC}$-axioms with qualified number restrictions.

b) Check with the Tableau Calculus the consistency of the concept $\text{Curry}$ with the axioms.
Exercise 5-3  Negation Normalform with Number Restrictions

The NNF of unqualified number restrictions is defined by the following rules:

- \( \neg(\text{atleast } n \ r) \mapsto \text{atmost } (n - 1) \ r \)
- \( \neg(\text{atmost } n \ r) \mapsto \text{atleast } (n + 1) \ r \)

Prove this using the semantics of the operators.

Exercise 5-4  \( \mathcal{ALC} \) with Number Restrictions

Consider the following facts:

- Farmer Krause owns at least 2 black horses.
- Farmer Krause owns at least 2 white horses.
- Farmer Krause owns at most 3 horses (with some colour)

What is the consequence for zebras?

Try to model the facts (including the consequences for zebras) as \( \mathcal{ALC} \)-formulae (with number restrictions).

Exercise 5-5  Repetition.: Tableau with Qualified Number Restrictions

Use the Tableau Calculus to check the consistency of:

a) \( \text{atmost } 2 \ r \top \sqcap \text{atleast } 3 \ r \ \psi \)

b) \( \text{atmost } 2 \ \text{has-Pet} \top \sqcap \exists \text{has-Pet Horse} \sqcap \exists \text{has-Pet Cat} \sqcap \exists \text{has-Pet Dog} \)

with the T-Box axioms: \( \neg(\text{Cat} \sqcap \text{Dog}), \neg(\text{Horse} \sqcap \text{Cat}), \neg(\text{Horse} \sqcap \text{Dog}) \).

Exercise 5-6  \( \mathcal{ALC} \) with Role Hierarchies

Consider a T-Box with

- the concept definitions and -axioms: \( A \sqsubseteq B \)
- and the following role axiom (Role hierarchy): \( r \sqsubseteq s \) und \( s \sqsubseteq t \)

Check the consistency of the formula \( \exists \ r \ A \sqcap \forall t \ \neg B \) with the T-Box using the Tableau Calculus.
Exercise 5-7  \textit{ALC mit Role Terms: Blonde Son}

Consider the following scenario:

- Daughters are Children.
- Sons are Children.
- Children are either Sons or Daughters.
- Karl’s sons are all blonde.
- Karl has a child named Thomas, and Thomas is no daughter.

a) Formulate these facts as \textit{ALC}-formulae using general role axioms. Use the roles has-Son, has-Daughter, has-Child and the single concept Blonde.

b) Prove with the Tableau Calculus that Thomas is blonde.

Exercise 5-8  \textit{ALC with Role Terms: Integration into the Tableau Calculus}

Integrate into the Tableau Calculus rules for treating the propositional aspects of role terms.

\textbf{Hint:} The modified rule for the universal quantifier ($x : \forall r \psi$) must do a case distinction:

- Case 1: $x \neg r y$
- Case 2: $x r y$ und $y : \psi$

How can we integrate T-Box axioms for concepts and roles?